Energy Saving Technological Progress in a Vintage Capital Model*

Discussion Paper version

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Abstract

Fossil fuel is an essential input throughout all modern economies. The reduced availability of this basic input to production, and the stabilization of greenhouse gases concentration—which requires reductions in fossil fuel energy use—would have a negative impact in GDP and economic growth through cutbacks in energy use. However, this trade-off between energy reduction and growth could be less severe if energy conservation is raised by energy saving technologies. Here we study this hypothesis and, in particular, the effect of a tax over the energy expenditure of firms as a way to promote investments in energy saving technologies. To do this we consider a general equilibrium model with embodied and exogenous energy saving technological progress in a vintage capital framework, where the scrapping rule is endogenous and linear simplifications are eliminated.

Keywords: Environment, Nonrenewable resources, Energy, Energy saving technological progress, Vintage capital.

Journal of Economic Literature: C68, H23, O31, O41, Q43

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1 Introduction

Fossil fuel—in particular petroleum and its refinery products—is an essential input throughout all modern economies. For example, in 1990, the United States (US) consumed 84.16 quadrillion Btu (British thermal units)[1]of energy from all sources; fossil fuels made up 71.98 quadrillion Btu, 85.5% of her total energy consumption. Fifty years ago the US was self-sufficient in her supply of petroleum; today she imports more than half of her petroleum and consumes 25% of the world supply. In particular, Petroleum dominates the transport sector of the energy consuming economy; this domination rose from 77% in 1949 to 97% in 1998. The increasing dependence on petroleum can already be detected in 1972, when the daily consumption was approximately 2.6 x 106 m^3 (cubic meters) (16.4 million bbr (barrels))[2] per day; by 1997, this number rose to 3.0 x 106 m^3 (18.6 million bbr) per day. Due to increasing growth of industry over the past twenty-five years, the average annual growth rate of US total petroleum consumption was 0.5%.

The importance of this input is also clear when we study the negative impact in economic activity of a rise in oil prices. In fact, eight of the nine recessions experienced by the US economy after the World War II were preceded by an increase in the oil price (Boucekkine and Pommeret (2002); see Brown and Yucel (2001) for a survey).

We can observe, as well, the high presence of fossil fuel in a world perspective (see Table 1). In 2000, the 41.3% of the world energy fuel was oil, and 63.7% together with natural gas. The Organization of the Petroleum Exporting Countries (OPEC) estimated a total world oil demand in 2000 around 76 million barrels per day; if world economic growth continues, crude oil demand will also rise to 90.6 m b/d (million of barrels per day) in 2010 and 103.2 m b/d in 2020, according to the OPEC's World Energy Model (OWEM) reference case figures. The International Energy Agency (IEA) confirmed these predictions with 76.5 m b/d in 2001 (annual change of 0.4%), 76.9 m b/d in 2002 (annual change of 0.5%) and 78 m b/d in 2003 (annual change of 1.5%) (IEA, Monthly Oil Market Report).

Despite of the importance of fossil fuel input we can point out two main reasons to promote solid policies about reduction of the current fossil fuel consumption.

First of all, fossil fuel—more precisely petroleum—is a *resource subject to exhaustion*. The average annual growth rate of world consumption of refined

[1]1 Btu = 0,2520 calories (cal); 1 Btu/minute=0,0176 kilowatts (kW).

[2]1 barrels (bbr) = 0,159 cubic meters (m^3).

Table 1: world energy fuel shares (*per cent*)

Energy	**1998**	**2000**	**2010**	**2020**
Oil	41.3	41.3	40.3	39.2
Gas	22.2	22.4	24.1	26.6
Solids	26.2	26.1	26.3	25.8
Hydro / Nuclear	10.4	10.3	9.3	8.5
Total	**100.0**	**100.0**	**100.0**	**100.0**

Source: OWEM Scenarios Report, March 2000

product between 1990 - 2001 was around 1.21%, while the average annual growth rate of world proven crude oil reserves along the same period was about 0.63% Furthermore, the OPEC estimated that OPEC's oil reserves are sufficient to last another 80 years at the current rate of production, while non-OPEC oil producers' reserves might last less than 20 years. However this could be a too optimistic forecast; indeed, the IEA in 1998 predicted that oil production would peak before 2015, so by 2020, demand will exceed supply by 17 m b/d.

A **second** reason—but not less important—is the so called *Greenhouse Effect*. The natural presence of greenhouse gases (GHGs) in the atmosphere is crucial for life in the surface of the Earth. Over the past century, human activities—specially what are related with fossil fuel consumption—have been releasing GHGs at a concentration unprecedented in geologic time (Ansuategi and Escapa (2002)). The Intergovernmental Panel on Climate Change (IPCC) observed an increase of GHGs' concentration around 30% since pre-industrial times. This upturn will eventually result in a global climate change over the course of the next few decades.

Therefore, even though fossil fuel is an essential input, the reduced availability of this basic element in production and the stabilization of greenhouse gases concentration would have a negative impact in economic growth, and development, through cutbacks in energy use (Smulders and Nooij (2003)). As a result we can ensure, a priori, a *trade-off between energy reduction and growth*.

However, this trade-off becomes less severe if energy conservation is raised by energy saving technologies. Carraro et al. (2003) observed that hypothesis, since new technologies can fundamentally alter the extent and nature of this trade-off. The effect of public policies on the development and spread of new technologies is among one of the crucial determinants of the success or failure of environmental management (see Löschel (2002), for a survey, and Jaffe et al. (2000)). Here we focus on the exhaustion problem of fossil fuel, considering the energy saving technological progress as a way to offset the negative effect of energy cutbacks. More precisely, a tax over the energy expenditure of firms is evaluated in our model as a way to promote investments in energy saving technologies.

In addition, other static comparative exercises—the effect of a variation in the disembodied technological progress, in the available energy supply and in the embodied energy saving technological progress—are developed here.

There is a growing evidence that energy saving technological progress has been significant in the last two decades. Newell et al. (1999) studied whether the increase in the energy cost in recent years induces energy savings innovation in the US; they concluded that the induced innovation hypothesis is very reasonable. Boucekkine and Pommeret (2002) studied the optimal pace of capital accumulation at the firm level when technical progress is energy saving. This model was based on one of the most accepted explanations of the inverse relationship between oil prices and economic activity (see Brown and Yucel (2001) for a survey), the so called *supply side effect*: rising oil prices are indicative of the reduced availability of basic inputs of productions. Baily (1981) observed that this supply side effect concerns the energy input itself but also and specially, the capital input. In fact, Baily argued that the productivity slowdown experienced by the US economy and the other industrialized countries after the first oil shock might well be due to a reduction in the utilization rate of capital, namely in the decrease of the effective stock of capital. The keywords for Baily are embodied technological change, obsolescence of capital goods, and the energy cost. Considering these ideas, Boucekkine and Pommeret (2002) developed a partial equilibrium model, at the firm level, to study the supply side effect depicted above in the presence of embodied energy saving technological progress. They modelled obsolescence by a vintage capital technology with an endogenous scrapping decision and complementarity between capital and energy inputs [3].

Our model is an extension of Boucekkine and Pommeret's (2002) contribution, to the general equilibrium case. Here we consider an exogenous technological progress embodied in the new capital goods, which are intro-

[3] In Baily's set-up, obsolescence is simply modelled through a decreasing effective output as capital ages, and there is no explicit scrapping decision. Moreover, in this model embodied technological progress makes capital good less productive over time.

duced in the economy through a vintage technology with endogenous obsolescence (scrapping) rule. In a general equilibrium model, Boucekkine et al. (1997) showed that the endogenous scrapping rule is constant with linear utility. Later, Boucekkine et al. (1998) considered the case of non linear utility function; they got an scrapping rule which converged non monotonically to its steady state value. The partial equilibrium model of Boucekkine and Pommeret (2002) generates a constant scrapping rule. Considering the general equilibrium case without linear simplifications, we assume a constant scrapping rule in the long run (Terborgh-Smith result); however this regularity may not be true along the transition. Eventually, this general equilibrium framework is also very interesting because it allows us to study the global effect of environmental policies over the economy, and its relation with the scarce energy supply and the expansion of energy saving technologies.

The paper is organized as follows. In section 2 we describe the model, the behavior of consumer and the rules that depicts both the optimal investment and the scrapping behavior of the firms. The balanced growth path (BGP) is presented in section 3, where we show the necessary conditions for its existence. Section 4 develops a static comparative analysis of our endogenous variables, along the BGP defined above. Finally, some concluding remarks are considered in section 5.

2 The Model

Following Boucekkine, Germain, and Licandro (1997), we consider an economy where the population is constant. There is only one final good (numeraire good) which can be assigned to consumption or investment. The final good is produced in a competitive market by a constant return to scale technology, which is defined over a continuum of inputs in the interval $[0, 1]$. One important difference with respect to the model by Boucekkine et al. (1996) is that, in our model, inputs are produced by mean of a non-linear technology. That technology is defined over vintage capital [4]. The input market is supposed monopolistically competitive to allow for a concave profit function in inputs sector [5]. Also we assume competitive labour market and exogenous available energy supply.

[4] We do not consider labor in order to simplify the model. Indeed, if we introduce labor in our model, it could be possible to normalize to unity the labor endowment of individuals; so that the per-capita labor supply is equals to one for all t.

[5] We need to avoid corner solutions because we consider symmetric equilibrium.

Household

Let us assume that the household considers the following standard intertemporal maximization problem with constant relative risk aversion (CRRA) instantaneous utility function (Boucekkine et al. (1996) considered a linear case)

$$\max_{c(t)} \int_0^\infty \frac{c(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt \tag{1}$$

subject to the budget constraint

$$\begin{aligned} \dot{a}(t) &= r(t)a(t) - c(t) \\ a(0) &\ \text{ given} \\ \lim_{t\to\infty} a(t)e^{-\int_0^t r(z)dz} &= 0 \end{aligned} \tag{2}$$

with initial wealth a_0, where $c(t)$ is per-capita consumption, $a(t)$ is per-capita asset held by and the interest rate $r(t)$ is taken as given for the household. θ measures the constant relative risk aversion, and ρ is the time preference parameter (it is assumed positive discount factor).

Defining the Lagrangian as

$$\mathcal{L}(t) = \frac{c(t)^{1-\theta}}{1-\theta} e^{-\rho t} + \lambda(t)(r(t)a(t) - c(t))$$

then we have the transversality condition

$$\lim_{t\to\infty} \lambda(t)a(t) = 0 \tag{3}$$

and

$$r(t) = \rho + \theta \frac{\dot{c}(t)}{c(t)} \tag{4}$$

Final Good Firm

The final good is produced competitively by a representative firm solving the following optimal profit problem

$$\max_{y_j(t)} \left\{ y(t) - \int_0^1 p_j(t)y_j(t)dj \right\}$$

where the per-capita production $y(t)$ is given by a constant elasticity of substitution (CES) production technology

$$y(t) = \left(\int_0^1 y_j(t)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

defined over a continuum of inputs $y_j(t)$ with $j \in [0,1]$. It is assumed a constant elasticity of substitution $\epsilon > 1$. Prices are given by

$$p_j(t) = \left(\frac{y_j(t)}{y(t)}\right)^{-\frac{1}{\epsilon}}$$

which comes from the standard monopolistic competition economy (Dixit and Stiglitz (1977)) and they are taken as given by the final good firm.

Input Firm

Producing in a monopolistically competitive market, the representative input-j firm maximizes her profits

$$\max_{y_j(t),i_j(t),T_j(t),p_j(t)} \int_0^{\infty} e^{-r(t)t} \left[p_j(t)y_j(t) - i_j(t) - e_j(t)Pe(t)(1+Z)\right] dt \tag{5}$$

subject to

$$y_j(t) = A\left(\int_{t-T_j(t)}^{t} i_j(z)dz\right)^{\alpha}, \quad 0 < \alpha \leq 1 \tag{6}$$

$$e_j(t) = \int_{t-T_j(t)}^{t} i_j(z)e^{-\gamma z}dz, \quad 0 < \gamma < r \tag{7}$$

$$p_j(t) = \left(\frac{y_j(t)}{y(t)}\right)^{-\frac{1}{\epsilon}} \tag{8}$$

with the initial conditions $i(t)$ given for all $t \leq 0$

$e_j(t)$ and $P_e(t)$ are respectively the demand and the price of energy, which are endogenous. Z is the expenditure energy tax defined by the government[6]. $i_j(t)$ is the investment of the representative input-j firm. The output and the price for input j are respectively represented by $y_j(t)$ and $p_j(t)$. The price of input j and the final good production per-capita, $y(t)$, are taken as given by the monopoly. The equation (6) is our non-linear technology defined over vintage capital. The energy demand is obtained by the equation (7). Here $\gamma > 0$ represents the rate of energy saving technological progress and $T_j(t)$ is the age of the oldest operating machines or scrapping age. Considering monopolistic competition, the inverse demand function is given by the equation (8).

Notice that new technology is more energy saving. Moreover it is important to observe that we assume complementarity between capital and

[6]It could be considered as a lump-sum tax.

energy (Leontieff technology). Certainly, each vintage $i_j(t)$ has an energy requirement $i_j(t)e^{-\gamma t}$. This assumption is unfailing from numerous studies; for instance Hudson and Jorgenson (1974), or Berndt and Wood (1975).

We define the capital stock

$$K(t) = \int_{t-T(t)}^{t} i(z)dz$$

and the optimal life of machines of vintage t

$$J_j(t) = T_j(t + J_j(t)) \tag{9}$$

Notice that $T_i(t) = J_i(t + T_i(t))$

Substituting (6)–(8) into (5), we have

$$\max_{i_j(t),T_j(t)} \int_0^{\infty} e^{-\int_0^t r(z)dz}$$
$$\left[(y(t)^{\frac{1}{\epsilon}} y_j(t)^{1-\frac{1}{\epsilon}} - i_j(t) - \int_{t-T_j(t)}^{t} i_j(z)e^{-r(z)}dz P_e(t)(1+Z) \right] dt$$

Let us consider the symmetric case, that is,

$$y_j(t) = y(t)$$
$$T_j(t) = T(t)$$
$$p_j(t) = 1$$

Then we can simplify our problem as follows

$$\max_{i(t),T(t)} \int_0^{\infty} e^{-\int_0^t r(z)dz} \left[y(t) - i(t) - e(t)P_e(t)(1+Z) \right] dt \tag{5'}$$

subject to

$$y(t) = A \left(\int_{t-T(t)}^{t} i(z)dz \right)^{\alpha}, \quad 0 < \alpha \leq 1 \tag{6'}$$

$$e(t) = \int_{t-T(t)}^{t} i(z)e^{-rz}dz, \quad \gamma > 0 \tag{7'}$$

$$J(t) = T(t + J(t)) \tag{9'}$$

$$i(t), \quad t \leq 0, \quad given$$

Taking (6')(7') into (5'), we have

$$\max_{i(t),T(t)} \int_0^\infty e^{-\int_0^t r(z)dz}$$
$$\left[A\left(\int_{t-T(t)}^t i(z)dz\right)^\alpha - i(t) - P_e(t)(1+Z)\int_{t-T(t)}^t i(z)e^{-r(z)}dz\right] dt$$

where $r(t)$ is given by (4).

By the definition of (9) (or (9')), the above maximization problem is equivalent to solve

$$\max_{i(t),T(t)} \int_t^{t+J(t)} e^{-\int_0^\tau r(z)dz}$$
$$\left[A\left(\int_{\tau-T(\tau)}^t i(z)dz\right)^\alpha - i(\tau) - P_e(\tau)(1+Z)\int_{\tau-T(\tau)}^t i(z)e^{-r(z)}dz\right] d\tau \tag{10}$$

From the first order condition (FOC) for $i(t)$, we get the *optimal investment rule*

$$\int_t^{t+J(t)} e^{-\int_0^\tau r(z)dz}\left[A\alpha\left(\int_{\tau-T(\tau)}^t i(z)dz\right)^{\alpha-1} - P_e(\tau)(1+Z)e^{-\gamma t}\right] d\tau \tag{11}$$
$$= 1$$

This expression is equivalent to

$$\int_t^{t+J(t)} \alpha A\left(\int_{\tau-T(\tau)}^\tau i(z)dz\right)^{\alpha-1} e^{\int_t^\tau r(z)dz}d\tau =$$
$$1 + \int_t^{t+J(t)} (1+Z)P_e(z)e^{-\gamma t}e^{\int_t^\tau r(z)dz}d\tau$$

where the left hand side is the discounted marginal productivity during the whole lifetime of the capital acquired in t; 1 is the marginal purchase cost at t, normalized to one; and the second term on the right hand side is the discounted operation cost at t.

The *optimal investment rule* establishes that firms should invest at time t until the discounted marginal productivity during the whole lifetime of the capital acquired in t exactly compensates for both its discounted operation cost and its marginal purchase cost at t.

From the FOC for $T(t)$, we have the *optimal scrapping rule*

$$A\alpha\left(\int_{\tau-T(\tau)}^t i(z)dz\right)^{\alpha-1} = P_e(t)(1+Z)e^{-\gamma(t-T(t))} \tag{12}$$

The *optimal scrapping rule* states that a machine should be scrapped as soon as its marginal productivity (which is the same for any machine whatever its age) no longer covers its operation cost (which rises with its age).

Here the marginal productivity is given by $\alpha A(\int_{t-T(t)}^{t} i(z)dz)^{\alpha-1}$, and $(1+Z)P_e(t)e^{-\gamma(t-T(t))}$ represents the operation cost.

By the definition of capital, we also get

$$A\alpha K(t)^{\alpha-1} = P_e(t)(1+Z)e^{-\gamma(t-T(t))}$$

that is

$$K(t) = \left(\frac{(1+Z)}{A\alpha} P_e(t)e^{-\gamma(t-T(t))}\right)^{\frac{1}{\alpha-1}} \tag{12'}$$

Summarizing, the (decentralized) equilibrium of our economy is characterized by the equations (2)–(4) (household side), the equations (6)–(9), the *optimal investment rule*, the *optimal scrapping rule*, and the following three additional equations to close the model: $c(t) + i(t) = y(t)$, $i(t) = \dot{a}(t)$ and $e(t) = e_s(t)$, the equilibrium condition in the energy market. $e_s(t)$ is the available energy supply[7]; in our model it is assumed exogenous.

3 Balanced growth path

We consider a quite extended idea of long run equilibrium, balanced growth path (BGP). This section defines our BGP, establishing later the necessary conditions for such an equilibrium. Notice that "Owing to mathematical complexities, the literature to date on vintage capital models hardly ventures into an analysis of the properties of the system when it is off the *golden-age equilibrium path*." (P. K. Bardhan (1966)). Moreover, the interior solution of our problem, observed by Howard C. Petith (1976), "may not exist, or if it exists may not be unique." Up to now, we are facing the same difficulties. In the following we consider that an interior solution exists for our economy. Instead of assuming the uniqueness, we note that our result do not depend on the uniqueness because the behavior of each BGP follows our analysis.

[7]The available energy supply is a flow (exogenous) variable, for example petrol or any other petroleum refinery product to generate energy. Here we do not explicitly treat an extraction sector.

3.1 Definition and necessary conditions

Let us define the BGP equilibrium as the situation where all endogenous variables grow at constant rate, which may be zero and/or different for each variable. Then from (4) and along the BGP we get

$$r(t) = \rho + \theta\gamma_C = \text{constant} = r \tag{13}$$

and

$$e^{-\int_t^\tau r(z)dz} = e^{-r(\tau-t)} \tag{14}$$

where γ_c is the growth rate of consumption.

A particular BGP equilibrium is explored here, where the scrapping age is constant in the long run

$$T(t) = J(t) = \overline{T}(< \infty) \tag{15}$$

Observe that this definition does not imply a constant scrapping rule for all $t \geq 0$; we only assume a constant scrapping rule along the BGP. Such an equilibrium (Terborgh-Smith result) is well known in economic literature[8].

Let us consider our economy along the BGP. Differentiating (11) and rearranging terms we obtain by (12)

$$(e^{\gamma T} - 1) - \frac{\gamma}{\gamma_{P_e} - r}(e^{(\gamma_{P_e} - r)J} - 1) = \frac{r}{(1+Z)\overline{P}_e} e^{\gamma - \gamma_{P_e} t} \tag{16}$$

where γ_{P_e} and $\overline{P}_e$ are respectively the growth rate and the level of the energy prices. The left hand side (LHS) is constant for any t in the balanced growth path, and the right hand side (RHS) is a function on t. So the equality holds if and only if

$$\gamma = \gamma_{P_e} \tag{17}$$

By the definition of $K(t)$ we have along the BGP that

$$\begin{aligned} K(t) &= \int_{t-\overline{T}}^{t} ie^{\gamma - iz}dz + \frac{i}{\gamma_i} e^{\gamma_i z}|_{t-\overline{T}}^{t} \\ &= \frac{i}{\gamma_i}(1 - e^{-\gamma_i \overline{T}})e^{\gamma_i t} \\ &= \overline{K}e^{\gamma_i t} \end{aligned} \tag{18}$$

[8]For example P.K. Bardhan (1966) and (1969), H.C. Petith (1976), Boucekkine et al. (1997) and (1998)

Indeed, the growth rate of investment (γ_i) and capital stock (γ_K) are equal. Moreover, by (12) and (17),

$$\begin{aligned} A\alpha K(t)^{\alpha-1} &= (1+Z)P_e(t)e^{-\gamma(t-\overline{T})} \\ &= (1+Z)\overline{P_e}e^{\gamma t}e^{-\gamma(t-\overline{T})} \\ &= (1+Z)\overline{P_e}e^{\gamma\overline{T}} \end{aligned} \tag{19}$$

Substituting (18) into (19) it yields

$$A\alpha(\overline{K}e^{\gamma_i t})^{\alpha-1} = (1+Z)\overline{P_e}e^{\gamma\overline{T}}$$

Hence, along the BGP,

$$e^{\gamma_i(\alpha-1)t} = \frac{(1+Z)\overline{P_e}e^{\gamma\overline{T}}}{A\alpha\overline{K}^{\alpha-1}} \tag{20}$$

From this equation we get the following result:

Proposition 1 *Assuming $0 < \alpha < 1$, the growth rates of investment and capital stock are equals to zero along the balanced growth path .*

Proof. It is easy to see that (20) holds if and only if $\alpha = 1$ and/or $\gamma_i = 0$. Otherwise the LHS goes to 0 when time t goes to ∞, but the RHS is a nonzero constant. Since we consider a constant long run optimal scrapping age, then we must have $\gamma_i = 0$.

In the case $\gamma_i = 0$ and $0 < \alpha < 1$ we get

$$i(t) = i = \text{constant}$$

$$K(t) = \int_{t-\overline{T}}^{t} i dz = i\overline{T}$$

♦

Notice that the case $\alpha = 1$ is not considered here because we want to avoid linear simplifications.

Since $\gamma > 0$ then we simply have

$$\begin{aligned} e(t) &= \int_{t-\overline{T}}^{t} ie^{-\gamma z dz} \\ &= \frac{i}{\gamma}e^{-\gamma t}(e^{\gamma\overline{T}}-1) = e_s(t) \end{aligned} \tag{21}$$

On the other hand, from (12), we have along the BGP that

$$A\alpha(i\overline{T})^{\alpha-1} = (1+Z)\overline{P_e}e^{\gamma t}e^{-\gamma(t-\overline{T})} = (1+Z)\overline{P_e}e^{\gamma\overline{T}}$$

So

$$i = \frac{1}{\overline{T}}\left(\frac{(1+Z)\overline{P_e}e^{\gamma\overline{T}}}{A\alpha}\right)^{\frac{1}{\alpha-1}} \tag{22}$$

From (16) and considering $\gamma = \gamma_{Pe}$ then

$$e^{\gamma\overline{T}} = 1 + \frac{r}{(1+Z)\overline{P_e}} + \frac{\gamma}{\gamma-r}(e^{(\gamma-r)\overline{J}} - 1) \tag{23}$$

Output along the BGP equals

$$y(t) = (K(t))^{\alpha} = A(i\overline{T})^{\alpha}$$

Moreover by the constraint

$$y(t) = i(t) + c(t)$$

along the BGP we have that the growth rate of consumption is zero ($\gamma_c = 0$) since $\gamma_i = \gamma_y = 0$, where γ_y is the growth rate of final good output. Hence

$$r(t) = \rho$$

Combining the above results we straightforwardly get the following proposition:

Proposition 2 *Along the balanced growth path, assuming* $0 < \alpha < 1$ *and* $\gamma < \rho$,

1. *the growth rate of investment, capital stock and final good output are equal to zero* ($\gamma_i = \gamma_y = \gamma_K = 0$)*;*
2. *the interest rate is constant and equal to the discount factor* ($r(t) = \rho = r^*$)*;*
3. *the growth rate of energy price* (γ_{P_e}) *is equal to the rate of energy saving technological progress* (γ)*;*
4. *the scrapping age is constant* ($T(t) = J(t) = \overline{T} = T^*$)*;*

5. *the inverse demand of available energy is given by*

$$P_e(t)^* = \frac{\overline{P}_e i^*}{\gamma}(e^{\gamma\overline{T}} - 1)\frac{1}{e(t)^*}$$

where $\overline{P}_e$ is the level of energy prices (i.e. $P_e(t)^* = \overline{P}_e e^{\gamma t}$);

6. *the available energy supply decreases in the constant rate γ (i.e. $e_s(t)^* = \overline{e}_s e^{-\gamma t}$).*

3.2 Observations

There are three observations to do here. **Firstly**, *our model has no growth in the long run.* This behavior is explained, on the one hand, by the assumption of decreasing returns to scale in the intermediate good technology; and on the other hand because here we get that both the scrapping age and the exogenous energy saving technological progress are not strong enough to overcome those decreasing returns. The reason is the following. Our framework considers CRRA instantaneous utility function; as a consequence, the interest rate is constant in the long run. Then, consistently with the Terborgh-Smith result, the scrapping age is also constant along the BGP. Taking the optimal investment rule in the long run

$$\alpha A e^{\rho t}\int_t^{t+\overline{T}}\left(\int_{\tau-\overline{T}}^{\tau} i(z)dz\right)^{\alpha-1} e^{-\rho\tau}d\tau = 1 + (1+Z)\overline{P}_e\frac{1}{\rho}(1-e^{-\rho\overline{T}})$$

it is straightforward that the discounted operation cost is constant because the effect of the energy saving technological progress (γ) is offset by the decreasing available energy supply. Hence, as the marginal purchase cost (1) is remaining constant, the investment has to be also constant along the BGP.

This is not a standard result. In the case of neoclassical models, we would have exogenous growth. For example, the models of Solow (1957) and Ramsey (1928), with exogenous technological progress, described economies which grew at the rate of both population growth and exogenous technological progress. Furthermore, considering linear technology, Boucekkine et al. (1997) and (1998) developed two vintage capital models, with exogenous technological progress, which generated growth and constant scrapping age along the BGP. On the contrary, our problem assumes non linear technology ($\alpha < 1$). We get now that the (exogenous) energy saving technological progress offsets the reduced availability of energy (non renewable resource)[9]; however, it is not strong enough to overcome the decreasing returns to scale. As a consequence, our economy has not growth along the BGP. This result is consistent with the partial equilibrium model of Boucekkine and Pommeret (2002), which depicts no growth along the BGP.

As a **second** observation notice that, in our model, *the investment only considers energy saving issues*. If we include other features—for example, R&D investments or abatement activities, taking into account pollution problems—optimal investment, and consequently our economy, might depict growth. Here we just focus on energy saving technological progress.

Finally, we have to point out the *necessity to assume a long run available energy supply* $e_s(t)^* = \overline{e}_s e^{-\gamma t}$ *to have BGP*. Here the exogenous term is $\overline{e}_s$, the level of available energy supply. This structure of the energy market implies that the energy prices increase at a constant rate γ, which is consistent with Boucekkine and Pommeret (2002). In their partial equilibrium model it is assumed an exogenous evolution of energy prices $P_e(t) = \overline{P}_e e^{\mu t}$. If the growth rate of energy prices (μ) is equal to the rate of energy saving technological progress (γ) then there exist a BGP; otherwise they got no BGP or unrealistic equilibrium situations.

4 Static comparative

In this section we study the static comparative of the model. We consider the effect of modifications in the parameters of the endogenous variables along the BGP. In our model this effect is mainly known by the behavior of the scrapping age and the investment. The performance of the scrapping age is described by the *optimal scrapping rule* in the long run (equation (22)). Similarly, the *optimal investment rule* (equation (11)) along the BGP

$$\alpha A(i^*\overline{T})^{\alpha-1}\frac{1}{\rho}(e^{\rho\overline{T}}-1) = 1 + (1+Z)\overline{P_e}\frac{1}{\gamma+\rho}(e^{(\gamma+\rho)\overline{T}}-1) \tag{24}$$

[9]Indeed, taking the optimal investment rule in the long run, without energy saving technological progress and decreasing available energy supply,

$$\alpha A e^{\rho t}\int_t^{t+\overline{T}}\left(\int_{\tau-\overline{T}}^{\tau} i(z)dz\right)^{\alpha-1} e^{-\rho\tau}d\tau = P_e(t)(1+Z)$$

it is easy to observe that, since the energy prices increases because of the reduced availability of energy, investment has to decrease for a constant scrapping age ($0 < \alpha < 1$).

establishes the behavior of the investment.

As in the long run the *optimal scrapping rule* (equation (22)) and the *optimal investment rule* (equation (24)) are functions of $i^*, \overline{T}$ and $\overline{P}_e$, we need one more equation to describe completely the behavior of both the scrapping age and the investment[10]. This third equation comes from the equilibrium condition of the energy market. The energy demand is given by the equation (21). As we assumed an exogenous long run available energy supply $e_s(t)^* = \overline{e}_s e^{-\gamma t}$ to have balanced growth path, then we get

$$\overline{e}_s = \frac{i^*}{\gamma}(e^{\gamma \overline{T}} - 1) \tag{25}$$

by equalizing energy demand and available energy supply. Therefore the values of optimal investment, optimal scrapping age and level of energy prices along the BGP are given by the equations (22), (24) and (25), which form a static (simultaneous) system of non-linear equations, taken the values of the parameters as given. Moreover, solving this system for different values of the parameters we can analyze the static comparative of our model. In Appendix B we include the parametrization, results and figures of our static comparative exercises.

4.1 Energy Expenditure Tax

Here we analyze the effect of an increase in the energy tax level of our economy. In addition, considering such a static comparative exercise, we can describe some of the differences between economies with unlike level of energy tax pressure. The purpose of this section is to describe the effects of an increase of Z over the optimal scrapping age ($\overline{T}$), the optimal investment (i^*) and the final good output (y^*).

A first approach is through a *pure analytical method*. From equation (25) we can obtain an expression for the investment as function of the scrapping age. Applying this to equation (22) (*optimal scrapping rule* in the long run) and differentiating that expression with respect to Z, we get a function $F_1(\frac{\partial \overline{T}}{\partial Z}, \frac{\partial \overline{P}_e}{\partial Z})$. To obtain the value of $\frac{\partial \overline{T}}{\partial Z}$ and $\frac{\partial \overline{P}_e}{\partial Z}$ we need a second equation $F_2(\frac{\partial \overline{T}}{\partial Z}, \frac{\partial \overline{P}_e}{\partial Z})$. This expression comes from a the differentiation of the *optimal investment rule* in the long run (equation (24)), after considerable manipulations. Although this method allow us to obtain the analytical value of the derivatives, we can not determine the sign of these for general values of the

[10] Observe that $\overline{P}_e$ (the level of energy prices) is an endogenous variable.

parameters, neither imposing restrictions over some of them.

So, let us consider an *alternative method* to the technique described above. This procedure is a combination between an analytical approach and the numerical solution of the static system of non-linear equations given by the expressions (22), (24) and (25).

By numerical methods—taking the empirical values of the parameters—we can solve that system of non-linear equations for different values of the energy expenditure tax $Z \in (0, 1)$. So, we can determine the sign of the derivatives simply by plotting $\overline{T}$, i^* and $\overline{P}_e$ against Z.

In particular, here we study the case of a quite high tax over the energy expenditure of firms. Bailey (2002) observed that taxes in the UK comprised 81.5% of total fuel prices. Following such an example, and consistently with the aim of the Kyoto Protocol, we assume a $Z = 0.80$. The results of the simulation suggests that an increase in the energy expenditure tax boots the optimal replacement age, and decreases both the optimal investment and the level of energy prices.

The inverse relation between the level of energy prices and the energy expenditure tax comes directly from the assumption of exogenous long run available energy supply. Considering the economy in the long run, if we increase that tax the available energy supply is not affected (notice that the economy is in the steady state) because it is exogenous and always decreasing in time[11]. However the energy demand is reduced since energy is now more expensive, for a fixed level of production and scrapping age. As a result the level of energy prices decreases.

About the other signs, we apply the negative relation between the level of the energy prices and the energy expenditure tax in the expressions obtained from the *pure analytical method*. In such a way, we can identify the positive and negative effects over the scrapping age and the investment in the long run. In the following we analyze the behavior of the scrapping age along the BGP.

4.1.1 Optimal Scrapping Age

Taking the *pure analytical method* and rearranging function $F_1(.)$, we get:

$$\frac{\partial \overline{T}}{\partial Z} = -\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} + \frac{\gamma}{1-\alpha}\right)^{-1} \frac{1}{1-\alpha}\left(\frac{1}{\overline{\overline{P}}_e}\frac{\partial \overline{P}_e}{\partial Z} + \frac{1}{1+Z}\right) \tag{26}$$

[11] Remember that our exogenous available energy supply is $e_s(t)^* = \overline{e}_s e^{-\gamma t}$ to have BGP. The exogenous element of that supply is the level of available energy $\overline{e}_s$

Here it is possible to distinguish two opposite effects of the energy expenditure tax over the scrapping age.

Direct effect: It is the effect of a modification in the energy tax over the scrapping age for a fixed level of energy prices:

$$\frac{\partial \overline{T}}{\partial Z}_{|\overline{P}_e fixed} = -\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} + \frac{\gamma}{1-\alpha}\right)^{-1} \frac{1}{1-\alpha}\frac{1}{1+Z}$$

The term $\frac{1}{1-\alpha}\frac{1}{1+Z}$ is always positive because $0 < \alpha < 1$ and $0 < Z < 1$. Considering $\gamma > 0$, $\rho > 0$ and $\overline{T} > 0$ it is easy to proof[12] that $(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}}-1} + \frac{\gamma}{1-\alpha})^{-1}$ is positive. Then the direct effect has a negative outcome over the scrapping rule i.e. $\frac{\partial \overline{T}}{\partial Z}_{|\overline{P}_e fixed} < 0$.

So, if the energy expenditure tax increases, the scrapping age is reduced for a fixed level of energy prices. The interpretation of this effect is clear. If the energy tax increases, the operation cost rises for a fixed level of energy prices. Consequently, firms decide to substitute earlier their equipment. We can verify this explanation taking the scrapping and investment rule in the long run. When the tax increases, firms can modify the decision about the scrapping age and investment. The net result is given by substituting the *scrapping rule* (equation (22)) into the *investment rule* (equation (24)). After some manipulations it yields:

$$(1+Z)\overline{P}_e \left[\left(\frac{1}{\rho} - \frac{1}{\gamma+\rho}\right) e^{(\gamma+\rho)\overline{T}} + \frac{1}{\rho} e^{\gamma \overline{T}} + \frac{1}{1+\rho}\right] = 1$$

When Z rises, firms compensate it by dropping the scrapping age for a fixed level of energy prices.

However, according to our simulation, this negative direct effect is overcome by an **indirect effect** of the energy expenditure tax over the scrapping age through the variation of the level of energy prices. This effect is described by the expression

$$-\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} + \frac{\gamma}{1-\alpha}\right)^{-1} \frac{1}{(1-\alpha)} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial Z}$$

from equation (26). As $\frac{\partial \overline{P}_e}{\partial Z}$ is negative[13], the indirect effect is positive. When the energy tax rises, the level of energy prices decreases. As a consequence,

[12] See Appendix A.

[13] Remember that if $0 < \alpha < 1$, $0 < Z < 1$, $\gamma > 0$, $\rho > 0$ and $\overline{T} > 0$ then $\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}}-1} + \frac{\gamma}{1-\alpha}\right)^{-1} > 0$

the operation cost of machines is reduced. Then, firms want to scrap later their equipment.

Summarizing, we can conclude the following. When the tax over the energy expenditure of firms rises, the operation cost of machines increases. Then, firms decide to replace earlier their equipment (**direct effect**). However this effect is overcome by the reduction in the level of energy prices which is produced also by the increasing of the tax (**indirect effect**). Hence, the net effect of an increase of the energy tax over the scrapping age is positive $\frac{\partial \overline{T}}{\partial Z} > 0$.

Therefore, our result gives a theoretical evidence that an increase of an already high energy expenditure tax does not induce earlier replacement of machines; this is because that tax also modifies the level of energy prices.

4.1.2 Optimal Investment

The investment is another important decision for the firms, together with the scrapping age of machines. Here we study how a tax over the energy expenditure of firms affects the investment choice i.e. $\frac{\partial i^*}{\partial Z}$

Differentiating the *scrapping rule* in the long run (equation (22)), and rearranging terms we get

$$\frac{\partial i^*}{\partial Z} = -i^* \left[\frac{1}{1-\alpha} \frac{1}{1+Z} + \left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial Z} + \frac{1}{1-\alpha} \frac{1}{\overline{\overline{P}}_e} \frac{\partial \overline{P}_e}{\partial Z} \right] \tag{26}$$

Here we can distinguish a **direct effect** of the tax over the investment and **two indirect effects**, through the scrapping age and the level of energy prices.

For a fixed scrapping age and level of energy prices, we get the **direct effect**:

$$\frac{\partial i^*}{\partial Z}_{|\overline{P}_e \text{ and } \overline{T} \text{ fixed}} = -i^* \left(\frac{1}{1-\alpha} \frac{1}{1+Z} \right) < 0$$

This effect is negative. If the tax increases, the operation cost of machines rises too. Then, firms decide to invest less for a fixed scrapping age and level of energy prices[14].

However, there are two additional indirect effects.

Indirect effect through the scrapping age: If the level of energy prices is fixed, the direct effect of the energy tax over investment might be

[14]See the *scrapping rule* in the long run with $\overline{P}_e$ and $\overline{T}$ fixed.

reduced, compensated or overcome by the effect of that tax through the scrapping age. This is represented by the term

$$-i^*\left(\frac{1}{\overline{\overline{T}}}+\frac{\gamma}{1-\alpha}\right)\frac{\partial\overline{T}}{\partial Z}$$

in equation (26). As in section 4.1 we showed that $\frac{\partial\overline{T}}{\partial Z}_{|\overline{P}_e fixed} < 0$, it is clear that this effect is positive. When the energy tax increases, firms decide to invest more because they replace earlier machines, for a fixed level of energy prices. This effect is easily observed by the *scrapping rule* in the long run (equation (22)).

Indirect effect through the level of energy prices: The total effect of a variation in the energy expenditure tax is much more complicate when we consider modifications in the level of energy prices. This indirect effect is given by the term

$$-i^*\frac{1}{1-\alpha}\frac{1}{\overline{\overline{P}}_e}\frac{\partial\overline{P}_e}{\partial Z}$$

As we showed in section 4.1, $\frac{\partial\overline{P}_e}{\partial Z}$ is negative. Then, this indirect effect is negative. When the energy tax increases, the level of energy prices decreases. So, firms invest more.

Considering both indirect effects, the net result is clear from the simulation. The energy tax has a negative effect over the investment through the level of energy prices; the positive indirect effect through the scrapping age is not strong enough to offset the effect of the energy prices.

Then, here we can conclude that the sign of $\frac{\partial i^*}{\partial Z}$ is negative. The energy expenditure tax has a negative **direct effect** over investment, because an increase of the tax rises the operation cost. This direct effect is reinforced by the negative **indirect effect through the level of energy prices**; the additional **indirect effect through the scrapping age** is not strong enough to offset the effect of the energy prices.

4.1.3 Final Good Output

The static comparative of the final good output is given directly from the static comparative of both the optimal scrapping age and the optimal investment considered before.

The $\frac{\partial y^*}{\partial Z}$ is given by the equation of the final good output in equilibrium

$y^* = A(i^*\overline{T})^\alpha$ Differentiating that expression with respect to Z we get

$$\frac{\partial y^*}{\partial Z} = \alpha y^* \left(\frac{1}{i^*}\frac{\partial i^*}{\partial Z} + \frac{1}{\overline{\overline{T}}}\frac{\partial \overline{T}}{\partial Z} \right)$$

From the previous sections, we know that $\frac{\partial \overline{T}}{\partial Z} > 0$ and $\frac{\partial i^*}{\partial Z} < 0$. According to our simulation, the effect of increasing an already high energy expenditure tax over the final good output is negative; the decrease of the optimal investment overcomes the later replacement of machines.

4.2 Disembodied Technological Progress

This section studies the effect of an increase in the technological level of all machines in our economy. Moreover we can apply this exercise to describe economies involving different levels of global technological progress. We have to analyze an increase in the disembodied technological progress to do that. The effects over the scrapping age, the investment and the final good out are considered in the following. Here we apply a similar strategy to the methodology developed before.

Taking logs and differentiating equations (22) and (25) with respect to A we get respectively

$$\frac{\partial i^*}{\partial A} = i^* \left[-\left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial A} - \frac{1}{1-\alpha}\frac{1}{\overline{\overline{P}}_e}\frac{\partial \overline{P}_e}{\partial A} + \frac{1}{A}\frac{1}{1-\alpha} \right] \tag{27}$$

$$\frac{\partial i^*}{\partial A} = -i^* \left(\frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \right) \frac{\partial \overline{T}}{\partial A} \tag{28}$$

4.2.1 Optimal Scrapping Age

Combining (27) and (28) it yields

$$\frac{\partial \overline{T}}{\partial A} = \left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \right)^{-1} \left(\frac{1}{A}\frac{1}{1-\alpha} - \frac{1}{1-\alpha}\frac{1}{\overline{\overline{P}}_e}\frac{\partial \overline{P}_e}{\partial A} \right) \tag{29}$$

In equation (29) we identify the **direct effect** of an increase in the disembodied technological progress (A) over the scrapping age ($\overline{T}$):

$$\frac{\partial \overline{T}}{\partial A}_{|\overline{P}_e fixed} = \left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \right)^{-1} \frac{1}{A}\frac{1}{1-\alpha}$$

This effect is positive (i.e. $\frac{\partial \overline{T}}{\partial A}_{|\overline{P}_e fixed} > 0$)[15]. When A increases, the marginal productivity of all machines rises too, and firms scrap later their machines. So $\overline{T}$ increases.

However, the net effect of A is a combination of the direct effect described above and the **indirect effect** of A over $\overline{T}$ thought the level of energy prices $(\overline{P}_e)$. This effect is given by

$$-\left(\frac{1}{\overline{\overline{T}}}+\frac{\gamma}{1-\alpha}-\frac{\gamma e^{\gamma\overline{T}}}{e^{\gamma\overline{T}}-1}\right)^{-1}\frac{1}{1-\alpha}\frac{1}{\overline{P}_e}\frac{\partial\overline{P}_e}{\partial A}$$

This indirect effect is negative because $\frac{\partial \overline{P}_e}{\partial A} > 0$. When A rises, the marginal productivity of all machines increases too. Therefore, firms replace later their equipment. The older a machine the greater its energy requirements; then the demand of energy rises for a fixed level of effective energy supply. As a consecuence, the level of energy prices $(\overline{P}_e)$ increases.

According to our parametrization, the net effect of an increase in the disembodied technological progress is to rise the replacement age. On the one hand, the higher A the higher the marginal productivity of all machines; therefore, firms scrap later their equipment (**direct effect**). On the other hand the level of energy prices increases, affecting negatively the scrapping age (**indirect effect**). However, the former effect is stronger than the direct effect. Then, firms decide to replace earlier their machines (i.e. $\frac{\partial \overline{T}}{\partial A} < 0$).

4.2.2 Optimal Investment

The effect of an increase in the disembodied technological progress over the optimal investment is given by equation (27)[16]. We can distinguish a **direct effect** of A over the optimal investment:

$$\frac{\partial i^*}{\partial A}_{|\overline{P}_e \ and \ \overline{T} \ fixed} = i^*\frac{1}{1-\alpha}\frac{1}{A} > 0$$

The higher A the higher marginal productivity of all machines. Consequently, firms invest more (see investment rule in the long run (24)) for a given scrapping age and level of energy prices.

[15]See appendix A.

[16]Observe that equation (28) directly gives us the $\frac{\partial i^*}{\partial A}$ as a function of $\frac{\partial \overline{T}}{\partial A}$. However the variation on $\overline{T}$ contains the variation of the level of energy prices. For that reason we use equation (27), which separates both effects.

The direct effect is reinforced by a decline in the scrapping age for a given level of energy prices (**indirect effect through the scrapping age**)

$$-i^*\left(\frac{1}{\overline{\overline{T}}}+\frac{\gamma}{1-\alpha}\right)\frac{\partial\overline{T}}{\partial A}>0$$

When A increases firms replace earlier their equipment, thus investment rises.

But we have to point out the **indirect effect through the level of energy prices**

$$-i^*\left(\frac{1}{1-\alpha}\frac{1}{\overline{\overline{P}}_e}\right)\frac{\partial\overline{P}_e}{\partial A}<0$$

This effect is negative because the higher A the higher the level of energy prices. However, this effect is not strong enough to offset the positive effects.

The net result is that an increase in the disembodied technological progress raises the optimal investment

$$\frac{\partial i^*}{\partial A}>0$$

4.2.3 Final Good Output

The final good output is given by $y^*=A(i^*\overline{T})^\alpha$. Taking logs and differentiating with respect to A we get

$$\frac{\partial y^*}{\partial A}=y^*\left[\frac{1}{A}+\alpha\left(\frac{1}{i^*}\frac{\partial i^*}{\partial A}+\frac{1}{\overline{\overline{T}}}\frac{\partial\overline{T}}{\partial A}\right)\right]$$

A priori, an increase in the disembodied technological progress rises the final good output since the marginal productivity of all machines grows (positive **direct effect**):

$$\frac{\partial y^*}{\partial A}_{|i^*\ and\ \overline{T}\ fixed}=y^*\frac{1}{A}>0$$

This is reinforced by a positive effect of A over i^* (**indirect effect though the optimal investment**):

$$\alpha\frac{1}{i^*}\frac{\partial i^*}{\partial A}>0$$

Both effects can not be offset by the negative **indirect effect of A through the scrapping age**:

$$\alpha y^*\frac{1}{\overline{\overline{T}}}\frac{\partial\overline{T}}{\partial A}<0$$

Hence, the net result of an increase in the disembodied technological progress is a rise of the final good output ($\frac{\partial y^*}{\partial A}>0$).

4.3 Available Energy Supply

In this section we study the effect of an increase in the level of available (exogenous) energy supply over the replacement, the investment and the output of our economy. This rising could be interpreted, for example, as the discovering of new oil wells or the establishment of new trade agreements with petroleum producer countries. Also we can apply this analysis to compare economies with different levels of available energy.

As in the previous section, but differentiating with respect to $\overline{e}_s$, we get

$$\frac{\partial i^*}{\partial \overline{e}_s} = i^* \left[-\left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial \overline{e}_s} - \frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial \overline{e}_s} \right] \tag{30}$$

and

$$\frac{\partial i^*}{\partial \overline{e}_s} = i^* \left(\frac{1}{\overline{e}_s} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \frac{\partial \overline{T}}{\partial \overline{e}_s} \right) \tag{31}$$

4.3.1 Optimal Scrapping Age

The effect of an increase in the level of available energy supply over the replacement age is given by combining (30) and (31)

$$\frac{\partial \overline{T}}{\partial \overline{e}_s} = \left(\frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{1}{\overline{\overline{T}}} - \frac{\gamma}{1-\alpha} \right)^{-1} \left(\frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial \overline{e}_s} + \frac{1}{\overline{e}_s} \right) \tag{32}$$

In equation (32) we observe the **direct effect** of $\overline{e}_s$ over $\overline{T}$:

$$\frac{\partial \overline{T}}{\partial \overline{e}_s}_{|\overline{P}_e \ fixed} = \left(\frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{1}{\overline{\overline{T}}} - \frac{\gamma}{1-\alpha} \right)^{-1} \frac{1}{\overline{e}_s} < 0$$

This is a negative effect[17]. It seems a strange result, because we expected an increase in the scrapping age due to the higher level of available energy supply. However, the reason is that we impose that the demand of energy has to be equal to the supply for each period. If we fix the level of energy prices, firms have to replace earlier their equipment (take equation (7') in the long run i.e. energy demand). For that reason we have to point out the **indirect effect** through the level of energy prices. This is described by

$$\left(\frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{1}{\overline{\overline{T}}} - \frac{\gamma}{1-\alpha} \right)^{-1} \left(\frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \right) \frac{\partial \overline{P}_e}{\partial \overline{e}_s} > 0$$

[17]See appendix A.

As the available energy supply increases, the level of energy prices decreases (i.e. $\frac{\partial \overline{P}_e}{\partial \overline{e}_s} < 0$). Then the indirect effect is negative.

Finally, given our parameters, the net effect is positive as we expected. A rise in the available energy supply reduces the level of energy prices (i.e. $\frac{\partial \overline{T}}{\partial \overline{e}_s} > 0$) . Therefore, firms decide to replace later their equipment.

4.3.2 Optimal Investment

Equation (31) describes the effect of an increase in the available energy supply over the optimal investment. In this case such an increase affects the optimal investment through two indirect effects.

If we fix the level of energy prices, the rise in the available energy supply reduces the optimal investment because the scrapping age is higher ($\frac{\partial \overline{T}}{\partial \overline{e}_s} > 0$). This is a negative **indirect effect through the scrapping age**. It is given by

$$\frac{\partial i^*}{\partial \overline{e}_s}_{|\overline{P}_e \ fixed} = -i^* \left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial \overline{e}_s} < 0$$

As we have shown in the previous section, the higher available energy supply the higher scrapping age. Then, for a fixed level of energy prices, when this supply increases, firms invest less because they decide to replace later their equipment.

However, there is an additional effect coming from the variation of the level of energy prices. This is a positive **indirect effect through the level of energy prices**:

$$\frac{\partial i^*}{\partial \overline{e}_s}_{|\overline{T} \ fixed} = -i^* \frac{1}{1-\alpha} \frac{1}{\overline{\overline{P}}_e} \frac{\partial \overline{P}_e}{\partial \overline{e}_s} > 0$$

When the available energy supply increases, the level of energy prices decreases for a fixed scrapping age. As a consequence, firms invest more because the operation cost it is reduced.

According to our parameters, the net effect is positive ($\frac{\partial i^*}{\partial \overline{e}_s} > 0$). When the available energy supply increases, the effect coming from a lower level of energy prices (**indirect effect through the level of energy prices**) overcomes the consequence of a higher replacement age (**indirect effect through the scrapping age**). Hence, firms invest more.

4.3.3 Final Good Output

The effect over the final good output straightforwardly comes from the behavior of the optimal scrapping age and the optimal investment. As in the previous, but differentiating with respect to the available energy supply, we get

$$\frac{\partial y^*}{\partial \overline{e}_s} = \alpha \left(\frac{1}{i^*} \frac{\partial i^*}{\partial \overline{e}_s} + \frac{1}{\overline{T}} \frac{\partial \overline{T}}{\partial \overline{e}_s} \right) > 0$$

As both effect are positive, the higher available energy supply the higher final good output.

4.4 Embodied Technological Progress

The effect of an increase in the energy saving technological progress, incorporated in new equipment, is analyzed here. As in the previous sections, we can apply this static comparative exercise to compare economies with different rates of energy saving technological progress.

Taking logs and differentiating equations (22) and (25) with respect to the embodied energy saving technological progress (γ)

$$\frac{\partial i^*}{\partial \gamma} = i^* \left(\frac{1}{\gamma} - \frac{\overline{T} e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \frac{\partial \overline{T}}{\partial \gamma} \right) \tag{33}$$

$$\frac{\partial i^*}{\partial \gamma} = i^* \left[- \left(\frac{1}{\overline{T}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial \gamma} - \frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial \gamma} - \frac{\overline{T}}{1-\alpha} \right] \tag{34}$$

we characterize this long run dynamic.

4.4.1 Optimal Scrapping Age

The effect of such a technological progress over the replacement decision of firms is described by combining equations (33) and (34):

$$\frac{\partial \overline{T}}{\partial \gamma} = \left(\frac{1}{\overline{T}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \right)^{-1} \left(\frac{\overline{T} e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{1}{\gamma} - \frac{\overline{T}}{1-\alpha} - \frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial \gamma} \right)$$

Avoiding the effect of the level of energy prices, an increase of γ reduces the scrapping age:

$$\frac{\partial \overline{T}}{\partial \gamma}_{|\overline{P}_e fixed} = \left(\frac{1}{\overline{T}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} \right)^{-1} \left(\frac{\overline{T} e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1} - \frac{1}{\gamma} - \frac{\overline{T}}{1-\alpha} \right) < 0$$

Comparing with the initial rate of energy saving technological progress, the higher γ the less energy requirements of new equipment. Then, firms decide to replace earlier their equipment for a fixed level of energy prices (see equation (36)).

However the replacement decision is also determined by the variation of the level of energy prices $\frac{\partial \overline{P}_e}{\partial \gamma} < 0$. When γ increases, new machines need less energy than before. Then, for a given scrapping age, the demand of energy decreases. As a consequence, the level of energy prices decreases too. This fall constitutes the **indirect effect** of γ through the level of energy prices:

$$-\left(\frac{1}{\overline{T}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}} - 1}\right)^{-1} \frac{1}{1-\alpha} \frac{1}{\overline{P}_e} \frac{\partial \overline{P}_e}{\partial \gamma} > 0$$

This is a positive effect[18]. Since the level of energy prices decreases, the operation cost falls and firms decide to replace later their equipment.

The combination of both effects gives us the net outcome of γ over the scrapping age. Given our parametrization, the effect of a decrease on the level of energy prices (positive **indirect effect**) is not strong enough to overcome the negative **direct effect**. Hence, the higher rate of (embodied) energy saving technological progress the lower replacement age ($\frac{\partial \overline{T}}{\partial \gamma} < 0$).

4.4.2 Optimal Investment

Equation (34) gives us the behavior of the optimal investment (i^*) in the face of a variation in the embodied energy saving technological progress (γ). Here we can distinguish a **direct effect** of γ over i^*:

$$\frac{\partial i^*}{\partial \gamma}_{|\overline{T} \ and \ \overline{P}_e \ fixed} = -i^* \frac{\overline{T}}{1-\alpha} < 0$$

For a fixed scrapping age and level of energy prices, when the rate of embodied energy saving technological progress increases, firms have incentives to invest more because the new equipment needs less energy to work. This is clear from equation (24) (*optimal investment rule in the long run*). Considering $\overline{T}$ and $\overline{P}_e$ unchanged, it is straightforward that the RHS rises when γ increases[19].To maintain the equality, investment has to increases to rise the

[18]From appendix A, changing $\overline{T}$ by γ and viceversa, it is straightforward that $\left(\frac{1}{\overline{T}} + \frac{\gamma}{1-\alpha} - \frac{\gamma e^{\gamma \overline{T}}}{e^{\gamma \overline{T}}-1}\right)^{-1} > 0$

[19]Differentiating RHS with respect to γ, for $\overline{T}$ and $\overline{P}_e$ fixed

LHS.

However, two positive **indirect effects** overcome the negative direct effect. A variation in γ modifies the replacement age ($\frac{\partial \overline{T}}{\partial \gamma} < 0$). This **indirect effect through the scrapping age** is described by

$$-i^* \left(\frac{1}{\overline{\overline{T}}} + \frac{\gamma}{1-\alpha} \right) \frac{\partial \overline{T}}{\partial \gamma} > 0$$

The higher γ the lower $\overline{T}$ (see the previous section). Then, firms decide to invest more.

This effect is reinforced by a second positive **indirect effect through the level of energy prices** ($\frac{\partial \overline{P}_e}{\partial \gamma} < 0$):

$$-i^* \frac{1}{1-\alpha} \frac{1}{\overline{\overline{P}}_e} \frac{\partial \overline{P}_e}{\partial \gamma} > 0$$

When the rate γ rises, the level of energy prices decreases. As a consequence, the operation cost of machines falls, inducing firms to invest more.

Summarizing, an increase in the rate of embodied energy saving technological progress boosts optimal investment ($\frac{\partial I^*}{\partial \gamma} > 0$).

4.4.3 Final Good Output

From the final good output in the long run, we get

$$\frac{\partial y^*}{\partial \gamma} = y^* \alpha \left(\frac{1}{i^*} \frac{\partial i^*}{\partial \gamma} + \frac{1}{\overline{\overline{T}}} \frac{\partial \overline{T}}{\partial \gamma} \right)$$

From the previous sections we know that $\frac{\partial i^*}{\partial \gamma} > 0$ and $\frac{\partial \overline{T}}{\partial \gamma} < 0$. According to our parametrization, the net effect of an increase in the rate of embodied technological progress is negative (i.e. $\frac{\partial y^*}{\partial \gamma} < 0$). The negative effect of the replacement age overcomes the positive effect of the optimal investment. As a consequence, the level of final good output falls.

This is an expected result. Boucekkine et al. (1997) and (1998) got a decreasing level of final good output after a positive shock on the (exogenous) embodied technological progress. In their models, the economy continued to grow; however, the levels—series without the trend—fell[20]. Moreover, as we

[20]See the second observation of Section 3.2

noted in the second observation of Section 3.2, this fall could be compensated by including additional elements in the investment. Indeed, if the increase in the embodied technological progress (γ) go with a rise in the disembodied technological progress (A), the final good output boosts[21].

Observe that, in any case, a fall in the level of final good output does not necessary implies a decrease in welfare. Gains in welfare could be finding during the transition to the long run equilibrium.

5 Concluding remarks

In this paper we studied the hypothesis proposed in the introduction. Even though fossil fuel is an essential input throughout all modern economies, both the reduced availability of this basic input and the stabilization of greenhouse gases, require reductions in fossil fuel energy use. This cutback generates a trade-off between energy reduction and growth. However, if energy conservation is raised by energy saving technologies, this trade-off might be less severe. In particular, we analyzed an important feature of this hypothesis; the employment of a tax over the energy expenditure of firms as way to promote investments in energy saving technologies. A general equilibrium model, with embodied exogenous energy saving technological progress and vintage capital technology, has been used to perform this study. In this model we also considered endogenous scrapping rule, without linear simplifications.

We focused our analysis on the long run consequences of modifications in a tax over the energy expenditure of firms. In addition, we studied other static comparative exercises; the effect of a variation in the disembodied technological progress, in the available energy supply and in the embodied energy saving technological progress. The methodology developed here was a combination of numerical and analytical methods.

We found that our model is very rich to capture the different elements that affect the long run behavior of our economy. In particular, we point out the usually forgotten subject of technology replacement, which plays an important role in issues about energy saving technological change. One important consequence of considering such a replacement effect is that an increase of an already high energy expenditure tax does not induce earlier replacement of machines; this is because that tax also modifies the level of energy prices. In contrast, policies to improve energy saving technological

[21] See Appendix B.

progress induce lower scrapping age.

Obviously, our analysis has some limitations. The main restriction here is the assumption of *exogenous* energy saving technological progress. It is clear that a tax over the energy expenditure of firms has effects over such a technological progress. Moreover, our model performs no growth in the long run; this is because both the exogenous energy saving technological progress and the constant scrapping age, do not overcome the decreasing returns to scale. Considering endogenous technological progress, sustainable growth might be generated. So, an important extension of our model could be the inclusion of *endogenous* energy saving technological progress. Some studies in the literature concerning to the importance of an endogenous technological progress in this kind of models are, for example, Carraro, Gerlagh and van der Zwaan (2003) or Buonnano, Carraro and Galeotti (2003). As we considered a general equilibrium model, a good possibility to implement this idea is through an R&D sector (see Löschel (2002) for a survey about technological change in economic models of environmental policy).

A second interesting extension could be the inclusion of a petroleum refinery sector, because we assumed exogenous available energy supply. Since the energy supply will be endogenous, the behavior of the energy prices would be more realistic, specially along the short run.

In general, both extensions would particularly improve the performance of our model to describe the transition to the BGP. Then, a welfare analysis could be developing in order to measure costs associated to the short run dynamic.

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Appendix A

Proposition. *If* $0 < \alpha < 1$, $0 < Z < 1$, $\gamma > 0$, $\rho > 0$ *and* $\overline{T} > 0$ *then*

$$\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma\overline{T}}}{e^{\gamma\overline{T}} - 1} + \frac{\gamma}{1-\alpha}\right)^{-1} > 0$$

Proof. The term inside the brackets is equal to

$$\frac{(e^{\gamma\overline{T}} - 1)(1-\alpha) - \gamma\overline{T}e^{\gamma\overline{T}}(1-\alpha) + \gamma\overline{T}(e^{\gamma\overline{T}} - 1)}{\overline{T}(e^{\gamma\overline{T}} - 1)(1-\alpha)}$$

If $0 < \alpha < 1$, $\gamma > 0$ and $\overline{T} > 0$ then the denominator is greater than zero[22].

About the numerator, rearranging terms we obtain that it is equal to

$$e^{\gamma\overline{T}}(1 - \alpha + \alpha\gamma\overline{T}) - (1 - \alpha + \overline{T}\gamma)$$

If we rename $\gamma\overline{T} = x$, the numerator is a function $f(x) = e^x(1 - \alpha + \alpha x) - (1 - \alpha + x)$. It is easy to see that $f(0) = 0$ and $f'(x) > 0$ because $x > 0$. Then the numerator is greater than zero.

As numerator and denominator are greater than zero, hence

$$\left(\frac{1}{\overline{\overline{T}}} - \frac{\gamma e^{\gamma\overline{T}}}{e^{\gamma\overline{T}} - 1} + \frac{\gamma}{1-\alpha}\right)^{-1} > 0$$

♦

Appendix B

In this paper we write a program for Gauss[23] to solve the static system of non-linear equations given by the expressions (22), (24) and (25). Its structure is simple. First, we assign values to the parameters of our model from the economic literature to get an optimal scrapping age and ratio optimal investment/gdp around, respectively, 16 years and 16%. As we want to analyze the effect of a variation in some exogenous variables, we generate a sequence of them. After, we solve the static system of non-linear equations by a standard Newton-Raphson algorithm. Finally, we plot the level of energy prices

[22] Observation: if $\gamma > 0$, $\rho > 0$ and $\overline{T} > 0$ then $e^{\gamma\overline{T}} > 1$ and $e^{\rho\overline{T}} > 1$

[23] GAUSS for Windows NT/95 Version 3.2.32

(P_e), optimal scrapping age ($\overline{T}$), optimal investment (i^*), final good output (y^*) and ratio $\frac{i^*}{y^*}$ against the different values of exogenous variables.

Table 2: Parametrization

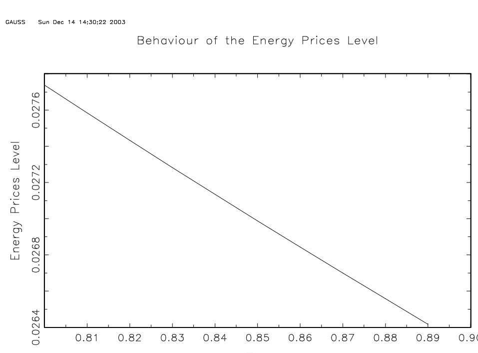
Parameter	ΔZ	ΔA	$\Delta \overline{e}_s$	$\Delta \gamma$
A	15	[14,15]	15	15
α	1/3	1/3	1/3	1/3
ρ	0.05	0.05	0.05	0.05
γ	6%	6%	6%	6%
$\overline{e}_e$	400	400	[400,410]	400
Z	[0.8,0.9]	0.8	0.8	0.8

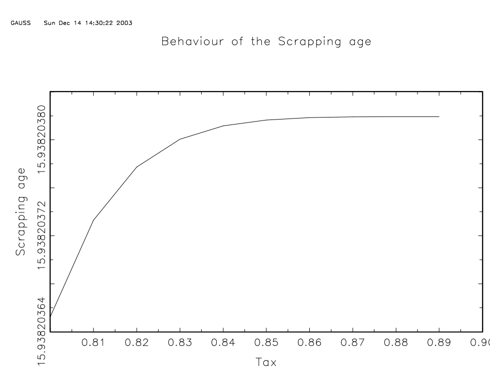
Table 3: Results

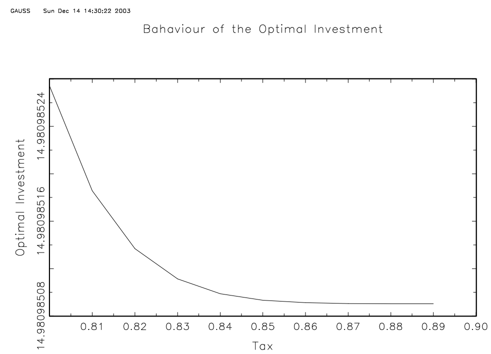
Parameter	ΔZ	ΔA	$\Delta \overline{e}_s$	$\Delta \gamma$	$\Delta \gamma \Delta A$
$\overline{P}_e$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\overline{T}$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
i^*	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
y^*	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$

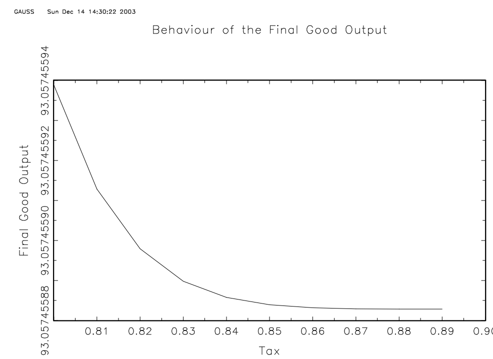
Figures:

Energy Expenditure Tax

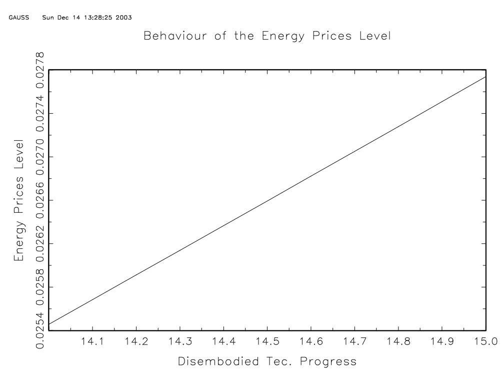


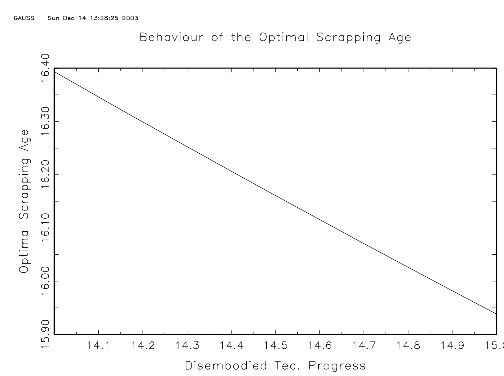


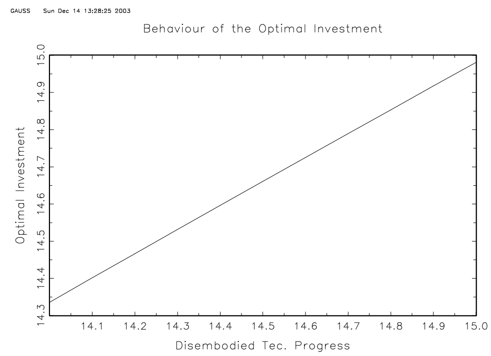


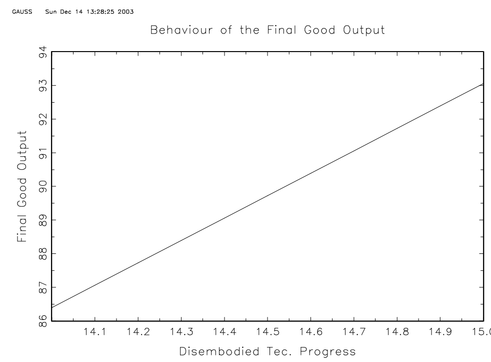


Disembodied Technological Progress

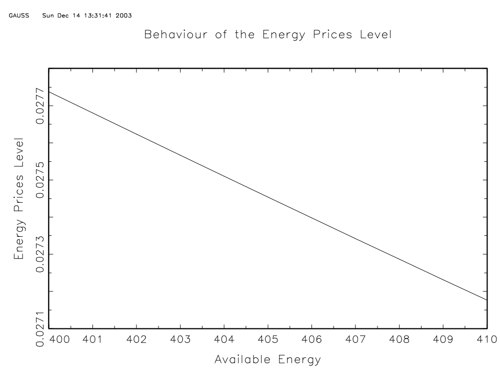


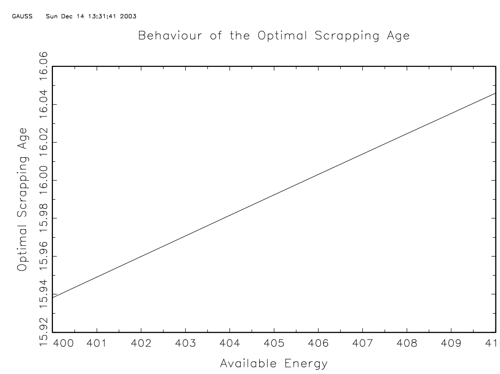


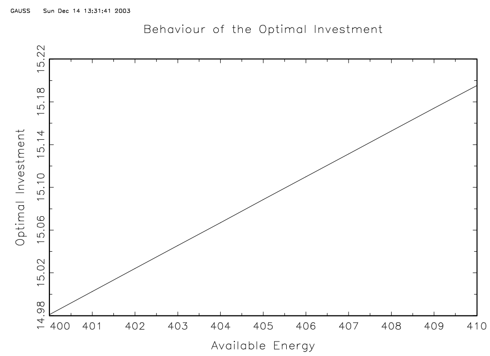


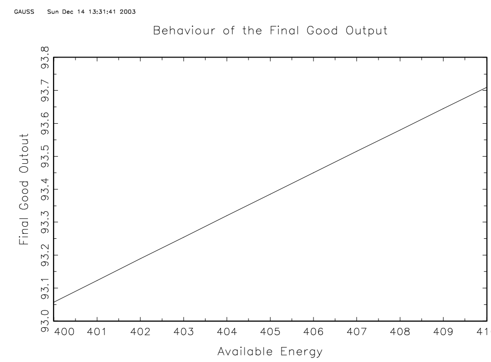


Available Energy Supply

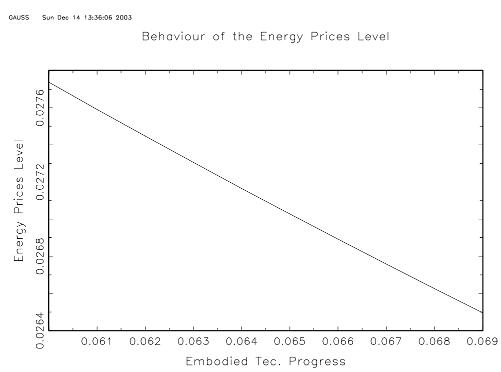


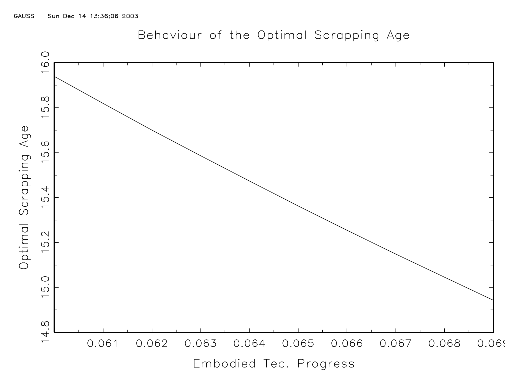


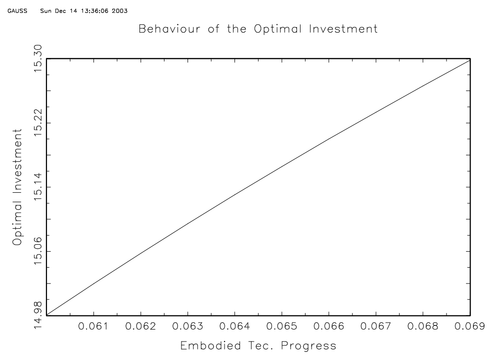


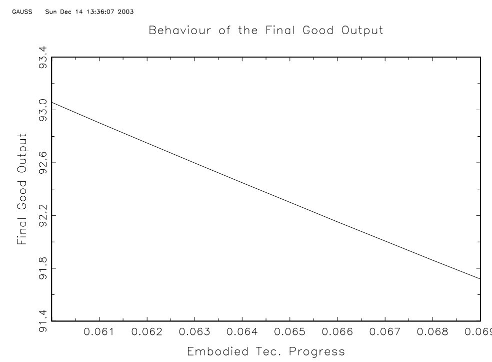


Embodied Technological Progress

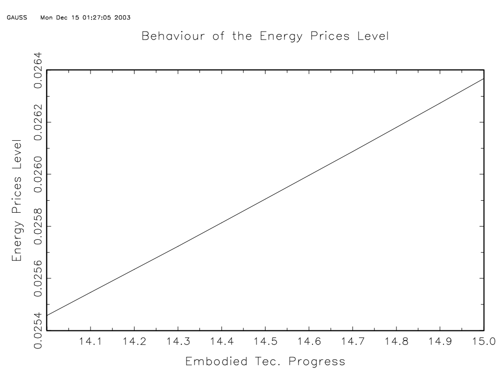


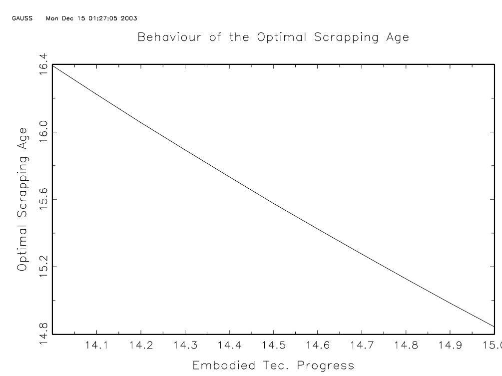


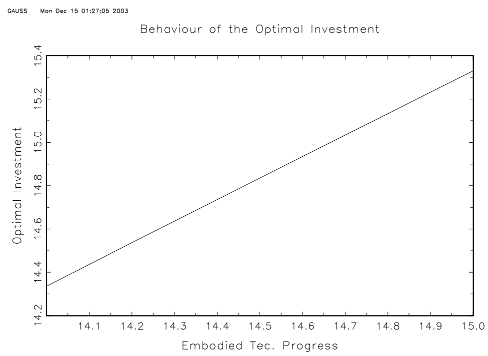




Embodied and Disembodied Technological Progress



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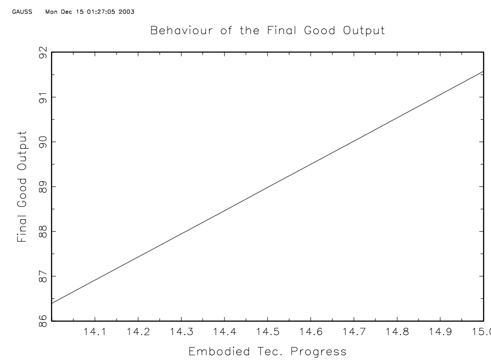
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Embodied Tec. Progress